# Other Harmony: Chords within Chords

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The important part of my lecture today will be devoted to research I have been doing since completing the book *Other Harmony*, a subject that I am calling "Chords within Chords", but first I want to acknowledge Javier Ruiz, who has been my copyist for many years, and who made an enormous contribution to the new book. In fact, *Other Harmony* would probably have been about half as long if Javier had not insisted that I must add a few pages here and there for passages that were not clear. He also advised me to add some of my Mathematica programs, to end each chapter with a few "exercises", which we call "continuation", to make a more detailed bibliography, and to insert a list of my compositions which employ these techniques, and all of this has added much to the comprehensibility and usefulness of the book, as well as to its length. At the same time he made many corrections, added numerous footnotes, added useful photos and illustrations, and generally edited the book, as well as designing the cover and the layout.

Also, before we put "Chords within Chords", I want to review some reactions I have already received from composer friends who have read the book.

Peter Ablinger wrote from Berlin that he was pleased to see someone give serious consideration to Josef Matthias Hauer, whom Ablinger considers extremely important. In researching my book last summer I found it necessary to acknowledge the contributions of Hauer and other theorists from the 1950s, who are seldom recognized, and it pleases me to have done this. If music theory is to advance, we must always consider the neglected theories of the past, as well as the accepted ones, and I am sure that Ablinger and I are not the only ones who will benefit from the chapters on Hauer, Schillinger, Slonimsky, and Obouhow.

Sergei Zagny wrote from Moscow that he was interested in harmony by sums, because his personal system of "magic stars" also works that way. I was familiar with Zagny's many pages of star music, which are a unique and personal kind of 12-tone music, and I even have the score, but since his harmonies by sums concern only four-note chords of a very specific type, it didn't occur to me to refer to that in my book. I will give an example of one of Zagny's stars here, however. Note that in this six-pointed star the first straight row, (2, 5, 9, 10), like the other five straight rows, all have the sum of 26, permitting the composer to form six four-note chords with the same height.


Zagny also said, however, that Forte was all wrong in saying that the major triad and the minor triad are equivalent, just because one is the inversion of the other. I already responded to this on pages 27-28 in my reference to Llorenç Balsach, who "was wanting to hear music as he has always heard it, rather than opening his ears to more objective listening." And as I said several times in the book, music is in any case too complex to be limited to only one explanation.

Carson Cooman wrote from Boston that he was pleased to see how many other composers had written with scales that did not repeat at the octave and sent me an example of such a scale that he had conceived 10 years earlier, constructed with transpositions of the first five notes of the Lydian mode:

Cooman said he had never used this in a composition, but perhaps he will want to do so now that he has seen the problem from the points of view of Slonimsky and Johnson.

Paul Epstein wrote from Philadelphia that he too was now trying to compose families of chords having the same sums, but that he wanted to try to do this with degrees of diatonic scales rather than with numbers of semitones. That strikes me as a strange notion, but he is a most sophisticated musician and I'm sure he knows what he is doing, and anyway, the important thing is that the book has stimulated his work.

I must also mention Nicolas Slonimsky's autobiography: *Nicolas Slonimsky: A Lite*, a book I'd never read until recently. After reading *Other Harmony* Yanik Miossec gave me a copy of this, and it makes wonderful reading, because Slonimsky seems to have known every interesting composer from Charles Ives to Frank Zappa. Most important for us here are Slonimsky's remarks on Joseph Schillinger and Nicolas Obouhow, as his comments tie together my three Russian theorists in a lovely way. Slonimsky hardly flatters these colleagues, but he talks about them in amusing and perhaps objective terms. (See Slonimsky pp. 79-80 and pp. 153-4). But now let me get on to "Chords within Chords."

You won't find the new word "homometric" in the dictionary, but it simply means chords that have the same sizes, the same interval content. Franck Jedrzejewski has studied homometric pairs a great deal, and it is indeed his fault that I began to compose pieces like *Intervals* and *La Princesse et les feuilles*, which are based solely on the all-interval tetrachords, in which the distances between the four notes include one minor second one major second, one minor third, one major third, one fourth and one tritone. Now I wanted to go further in this direction, and since I am now working on pieces for seven instruments, I wondered if I might make a movement with one of the seven-note homometric pairs included in Allen Forte's list in *The Structure of Atonal Music*. (It should be noted in parentheses that this important man, with whom 1 studied at Yale, just passed away a year or so ago.) I decided to work with the 48 chords derived from the pair Forte 7-Z18 and Forte 7- Z38, which I computed in this way:

Do[ch[i] = Mod[{0, 1, 2, 3, 5, 8, 9} + i, 12];

ch[12 + i] = Mod[{0, 1, 4, 6, 7, 8, 9} + i, 12];

ch[24 + i] = Mod[{0, 1, 2, 4, 5, 7, 8} + i, 12];

ch[36 + i] = Mod[{0, 1, 3, 4, 6, 7, 8} + i, 12], {i, 0, 11}]

Output:

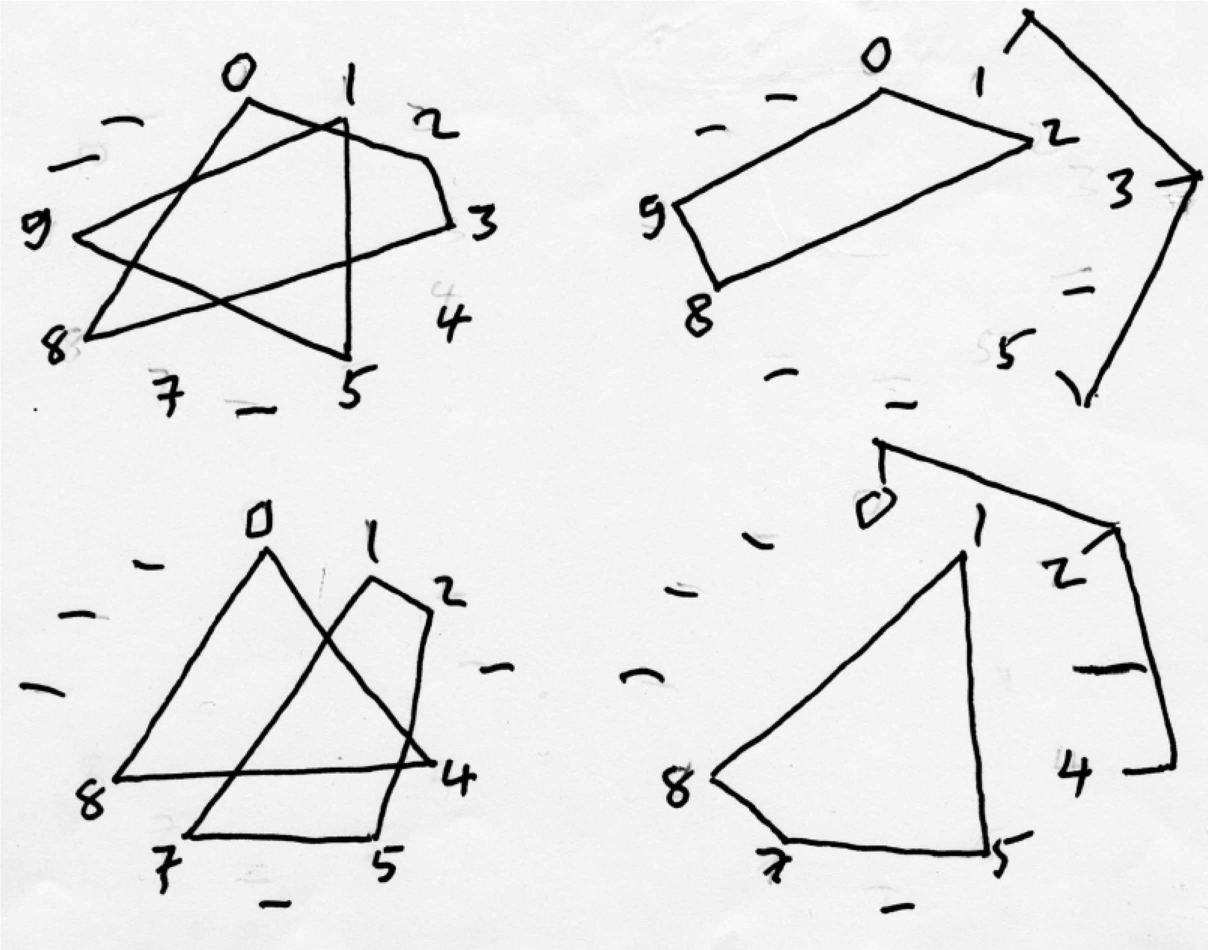
| | |
|---|---|
| 0 {0,1,2,3,5,8,9} | 24 {0,1,2,4,5,7,8} |
| 1.{1,2,3,4,6,9,10} | 25 {1,2,3,5,6,8,9} |
| 2 {2,3,4,5,7,10,11} | 26 {2,3,4,6,7,9,10} |
| 3 {0,3,4,5,6,8,11} | 27 {3,4,5,7,8,10,11} |
| 4 {0,1,4,5,6,7,9} | 28 {0,4,5,6,8,9,11} |
| 5 {1,2,5'6'7'8,10} | 29 {0,1,5,6,7,9,10} |
| 6 {2,3,6,7,8,9,11} | 30 {1,2,6,7,8,10,11} |
| 7 {0,3,4,7,8,9,10} | 31 {0,2,3,7,8,9,11} |
| 8 {1,4,5,8,9,10,11} | 32 {0,1,3,4,8,9,10} |
| 9 {0,2,5,6,9,10,11} | 33 {1,2,4,5,9,10,11} |
| 10 {0,1,3,6,7,10,11} | 34 {0,2,3,5,6,10,11} |
| 11 {0,1,2,4,7,8,11} | 35 {0,1,3,4,6,7,11} |

| | |
|---|---|
| 12 {0,1,4,6,7,8,9} | 36 {0,1,3,4,6,7,8} |
| 13 {1,2,5,7,8,9,10} | 37 {1,2,4,5,7,8,9} |
| 14 {2,3,6,8,9,10,11} | 38 {2,3,5,6,8,9,10} |
| 15 {0,3,4,7,9,10,11} | 39 {3,4,6,7,9,10,11} |
| 16 {0,1,4,5,8,10,11} | 40 {0,4,5,7,8,10,11} |
| 17 {0,1,2,5,6,9,11} | 41 {0,1,5,6,8,9,11} |
| 18 {0,1,2,3,6,7,10} | 42 {0,1,2,6,7,9,10} |
| 19 {1,2,3,4,7,8,11} | 43 {1,2,3,7,8,10,11} |
| 20 {0,2,3,4,5,8,9} | 44 {0,2,3,4,8,9,11} |
| 21 {1,3,4,5,6,9,10} | 45 {0,1,3,4,5,9,10} |
| 22 {2,4,5,6,7,10,11} | 46 {1,2,4;5,6,10,11} |
| 23 {0,3,5,6,7,8,11} | 47 {0,2,3,5,6,7,11} |

One might think that this is a balanced incomplete black design (12, 7, 16), since there are 12 elements, partitioned into subsets of seven, with the pairs coming 16 times within the 48 chords, but that is not the case. The vector is 434442, which means that distances between notes include 4 minor ·seconds, 3 major seconds, 4 minor thirds, and so on. Most of the intervals are found four times in each chord, but the major second occurs only three times and the tritone only twice, so it is an unbalanced design, and not one of the balanced ones that the mathematicians study. But of course, it is nonetheless a neat symmetrical system, which can be good for making neat symmetrical music.

One can not hear much difference between one seven-note chord and another, especially when the notes are all in the same range, so I thought it would be interesting for my septet to separate the four wind instruments (2 fl ob cl) and the three strings (2 vn vla). Can one find nice neat ways to divide these homometric seven-note chords into four-note chords for the winds and three-note chords for the strings?

lndeed there are. Let's look at the formation of the two basic seven-note chords as represented on a circle of 12 notes. I noticed that in half of the chords three of the four major thirds always come as an augmented triad, as one can see in the equilateral triangles of this diagram, and in the other half the three major seconds always come together in a do-re-mi form, as one can see around the edges. Curiously, the four remaining notes will always be all-interval tetrachords, which contain each interval once: (0, 1, 4, 6) or (0, 1, 3, 7).




This investigation already raises some interesting questions for a mathematician. Given subsets of a pair of chords with the same intervallic content, will their complements always have the same intervallic content as well? Why? Under what conditions will this intervallic content be an all-interval tetrachord? Can one propose a theorem that defines the particular cases?

Since I am not a mathematician I can't even imagine how one might answer such questions. My job is only to make music, and that means to take this observation and simply write it out. I could simply give my simple triads to the three strings and let the four winds play the all-interval tetrachords that would complete my 48 seven-note chords. I wrote this program to find the four three-note chords that go with the basic all-interval tetrachord (0,1,4,6).

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Do[If[Intersection[ch[i], {O, 1, 4, 6}] == {O, 1, 4, 6},      Print[i,” “,complement[ch[i], {0,1, 4, 6}]]], {i, 0, 47}]
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Output:

4 {5,7,9}

12 {7,8,9}
35 {3,7,11}
36 {3,7,8}

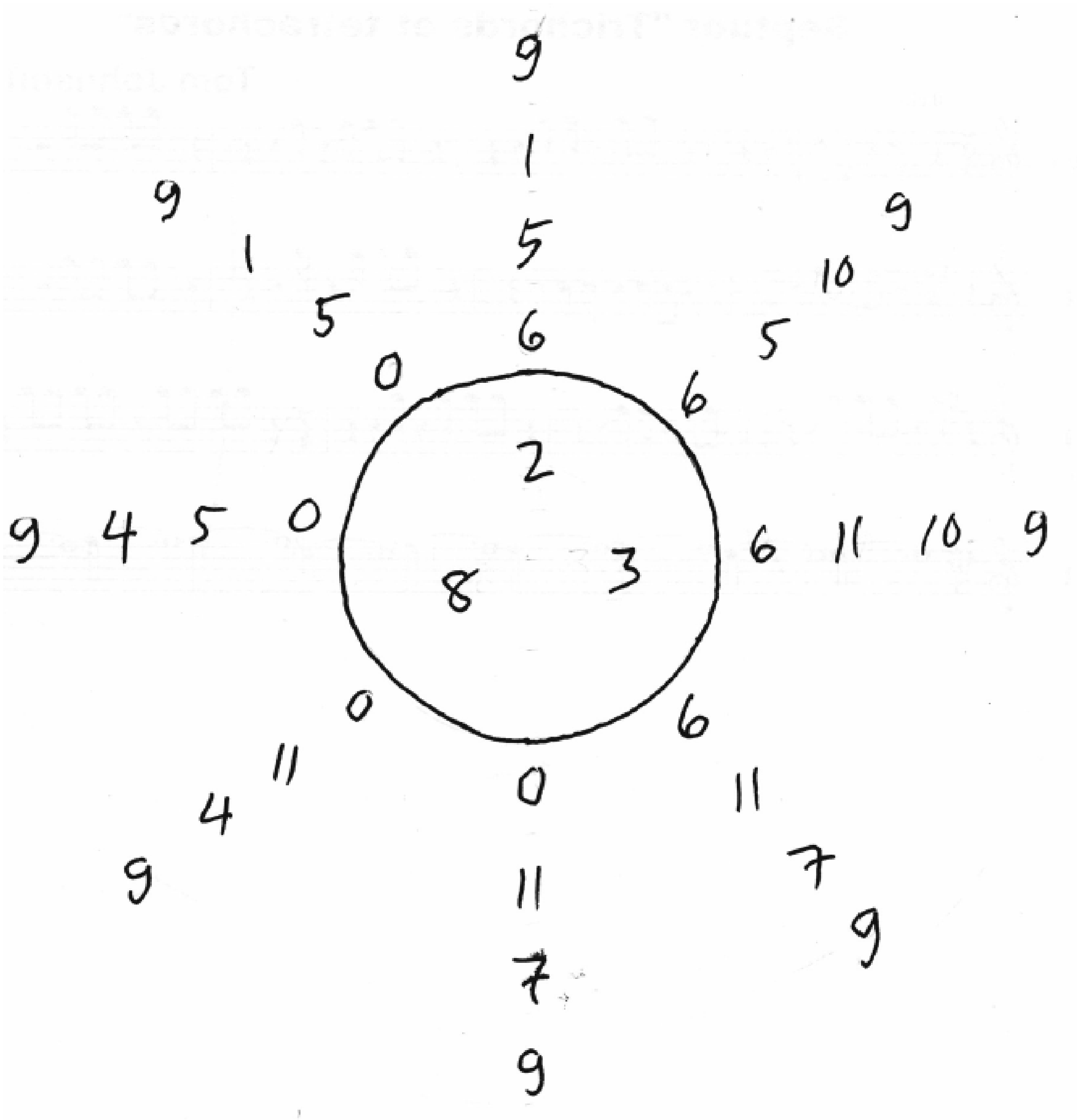
And this program determines the four three-note chords that go with the other all interval set (0,1,3,7).

```
Do[If[Intersection[ch[i], {1, 2, 5, 7}] == {1, 2, 5, 7},     Print[i, " ", Complement[ch[i], {1, 2, 5, 7}]]]], {i, 0, 47}]
```

Output:

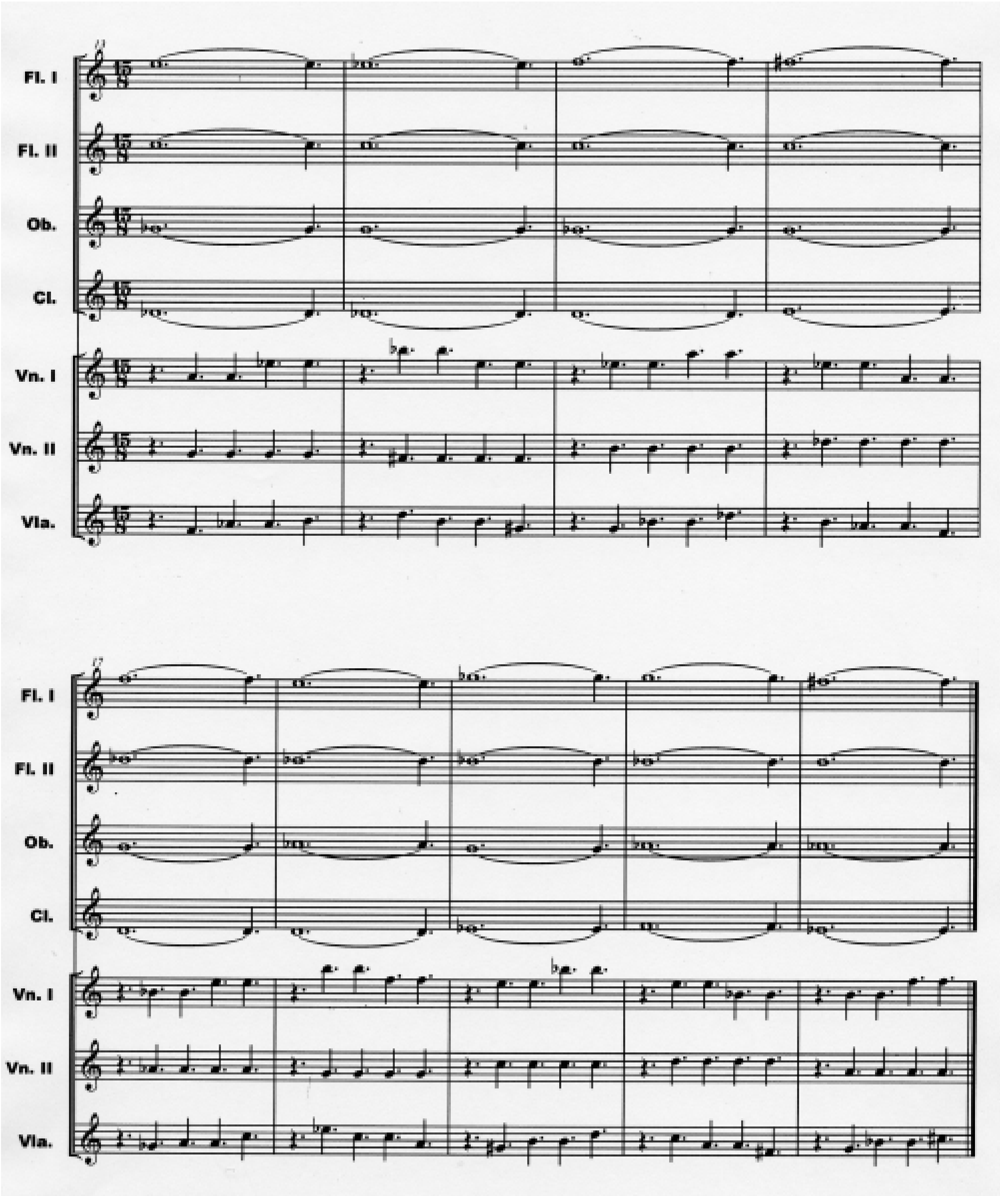
5 {6,8,10}
13 {8,9,10}
24 {0,4,8}
37 {4,8,9}

This seemed obvious and simple, but as I tried to figure out what sequence to use, and whether to work with the inversions of the all-interval tetrachords or not, I wasn't at all sure how to do this. I ran another program to determine which of my seven-note chords were most similar to one another, and I found that each chord had a maximum similarity with two other chords, and together they formed neat cycles of eight chords·. My graphic solution was to place three unchanging notes in a central circle, the two notes ·that go with them to form an all-interval tetrachord in a circle around that, and the three remaining notes that complete the eight chords in the three outer circles. In other words, the diagram shows how eight chords, all members of our homometric seven-note set, and all containing notes 2, 3, and 8, cycle in such a way that only one note changes with each new chord. I'll just show one of the six circles here:




Note that the idea of ordering chords in such a way that only one note changes with each progression comes directly from Hauer. You can find many examples of this in his piano pieces, and it is something that can never happen in serial music. If you are obliged to use all twelve notes before repeating one, you can not make gradual progressions of this sort. This piece, to be called *Septuor "trichords et tetrachords*," has not been performed yet, and won't be for some time, as it is part of a series of seven septets to be premiered by the Ensemble Offrandes in Le Mans and Alençon, probably early in 2017. I can, however, show you a couple of pages from my score, so you can see how I distributed the chords within the chords, that is, how the tetrachords in the winds combined with the trichords in the strings to make up the 48 seven-note chords of the homometric set.


But since this is a Symmetry Festival, let's talk about symmetry. This is a subject that has interested me for a long time, and it was around 1980 that I wrote my *Symmetries* for piano four hands. At that time I thought that symmetry was just a matter of mirroring the left side on the right side, and sometimes mirroring the top at the bottom, and that's the way I wrote these 49 pieces. But of course, symmetry goes much further than this simple visual model, and mathematicians know much more about commutative systems, symmetrical groups, and much more. But already here in these diagrams, one can see symmetries much more subtle than simply mirror images.




One of the chapters in *Other Harmony* is entitled "Equal and Complete", and I think these two qualities have a lot to do with symmetry. My composition *Septet "Trichords and Tetrachords"* has a nice balance, a symmetry, simply because the 48 chords all come from the same homometric set. They all have the same interval content, and all have a similar sound, so there is equality in the elements, but there is also completeness, since I rigorously included all 48 chords, all the combinations of the trichords and the tetrachords. So I am going well beyond mirror symmetry, but I want to understand more about symmetry and to go further in this direction.

As you can imagine, one of the main reasons I wanted to come to Vienna for this meeting, was precisely to learn more about all of this.